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- 05.00.00 - Texnika fanlari
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Metallurgiya. Aviasiya texnikasi
- 05.03.00 - Asbobsozlik, metrologiya va axborot-o'lchov
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- 05.04.00 - Radiotexnika va aloqa
- 05.05.00 - Energetika va elektrotexnika.
Qishloq xo'jaligi ishlab chiqarishini elektrlashtirish
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- 05.06.00 - To'qimachilik va yengil sanoat materiallari
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- 05.07.00 - Qishloq xo'jaligi ishlab chiqarishini
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- 05.08.00 - Transport
- 05.09.00 - Qurilish
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Mundarija

1	Oybek Achilovich Kayumov, Nazokat Rashitovna Kayumova. A Safety-Constrained Speaker-Adaptive Algorithm for Foreign Language Pronunciation Assessment in Correctional Education	6
2	Хамиджонов Шермурод Хомиджон ўғли. Namangan viloyatida sanoat tarmoqlari joylashuvining oʻziga xos xususiyatlari va rivojlanish tendensiyalari	10
3	Fakhriddin A. Boboyev, Malokhat U. Ilkhamova, Shakhruzbe O. Ozodov. Optimizing Overlock Seam Parameters for Joining Footwear Uppers to Nonwoven Strobel Insoles	14
4	Nuriddin Tojiyev, Shohruh Safarov, Behzod Tahirov, Otabek Abdullayev. Thermo-Magneto-Electro-Elastic Nonlinear Bending of Anisotropic Plates of Complex Shape	18
5	Yusufov Khakim, Khamidov Ilyosbek. Investigation of the strength properties of polymer-modified asphalt concrete mixtures	24

Thermo-Magneto-Electro-Elastic Nonlinear Bending of Anisotropic Plates of Complex Shape

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Abstract—This paper proposes a mathematical model and a computational algorithm for the nonlinear deformation of anisotropic plates of complex shape subjected to coupled thermal, electric, magnetic, and mechanical fields. The model is based on the Hamilton-Ostrogradsky variational principle and the Kirchhoff-Love hypothesis, together with the von Karman geometric nonlinearity describing large deflections; the coupled effects of the temperature, electric, and magnetic fields in the anisotropic medium are incorporated into the constitutive equations through the Duhamel-Neumann law, the Lorentz force, and Maxwell's equations. An arbitrary multiply connected plate domain and mixed boundary conditions are described by a single analytical expression by means of the R-function method, and a solution structure that exactly satisfies the boundary conditions is constructed. Spatial discretization is performed using the Bubnov-Galerkin method and Gaussian quadrature, while time integration is carried out using the implicit Newmark scheme combined with an iterative algorithm based on an equivalent stiffness matrix for the nonlinear terms. Numerical experiments were performed for a rigidly clamped graphite-epoxy plate under a distributed mechanical load, a magnetic field, and a temperature change. The results show that linear theory overestimates the maximum deflection by 16.46 percent compared with the nonlinear solution, which converged in eight iterations, owing to the membrane stiffening effect. The maximum deflection is inversely proportional to the cube of the plate thickness, with a correlation coefficient of 0.996. The proposed approach is suitable for engineering calculations of composite structural elements.

Keywords—*R-function method, anisotropic composite plate, thermo-magneto-elasticity, geometric nonlinearity, Duhamel-Neumann law, Bubnov-Galerkin method, Newmark scheme*

INTRODUCTION

The mechanics of modern aerospace engineering, automotive engineering, and smart structures requires

structural elements with minimal mass and high specific stiffness, which necessitates the extensive use of anisotropic composite materials. In practice, such elements rarely have a solid geometry: they contain L-, T-, I-, U-shaped, and star-shaped technological cutouts, which makes their domains multiply connected. At the same time, they operate under the simultaneous action of several coupled physical fields, namely mechanical, thermal, electric, and magnetic fields. Stress concentrations arise at the corner points of the cutouts, while the interaction of the fields alters the effective stiffness and dissipative properties of the structure. Therefore, adequate modeling of the stress-strain state of such objects is a pressing scientific and practical problem.

The main difficulty in solving problems for plates of complex shape lies in constructing coordinate functions that exactly satisfy the conditions prescribed on the domain boundary. Variational methods based on classical trigonometric and polynomial bases are effective only for canonical domains, whereas the finite element and finite difference methods require a very fine mesh near corner points and do not provide an analytical structure of the solution. The R-function method makes it possible to describe a boundary of arbitrary complexity by a single analytical expression and to construct a solution structure that exactly satisfies all boundary conditions. As a result, the sought solution is directly linked to the geometric features of the object.

The aim of this work is to develop a mathematical model and a computational algorithm for the problem of thermo-magneto-electro-elastic nonlinear deformation of anisotropic plates of complex shape based on the R-function method and the Bubnov-Galerkin variational approach, and to investigate the influence of the interaction of physical fields on deformation processes. At the initial stage of the study, the reliability of the developed algorithm and software was verified using the

semi-analytical Bubnov–Galerkin method and classical benchmark problems. At the main stage, the following were analyzed: geometrically nonlinear effects manifested at large displacements; the equivalent stiffness and damping properties arising from magneto-elastic interaction; and the influence of the macro-topology of the structure, its thickness h , and external boundary conditions on global stiffness and stability.

LITERATURE REVIEW

The theoretical foundations of the mechanics of anisotropic and composite plates were established in the works of S. A. Ambartsumyan [1] and J. N. Reddy [2]. Refined theories accounting for transverse shear and rotary inertia were substantiated by R. D. Mindlin [3], while the geometrically nonlinear statics and dynamics of flexible plates were systematically studied by A. S. Volmir [4], [5]. The theory of thermoelasticity, including the Duhamel–Neumann relations, is presented in the monograph by W. Nowacki [6]. The magnetoelasticity of thin shells and plates has been investigated in depth in the works of S. A. Ambartsumyan, G. E. Bagdasaryan, and M. V. Belubekyan [7], F. C. Moon [8], and H. Parkus [9], and was further developed in studies by C. Miehe, A. K. Noor, J. Tani, I. T. Selezov, R. Ansari, Z. Kuang, and others. In these studies, Maxwell’s equations and the Duhamel–Neumann law were applied in conjunction with linear and von Kármán nonlinear theories. Related coupled problems have also been addressed recently, including the nonlinear stability of magneto-electro-thermo-elastic beams [10], the transient behavior of functionally graded plates in a thermal environment [11], coupled wave propagation across a magneto-electro-thermo-elastic interface [12], the dynamic thermoelectroelasticity of a rigidly fixed circular plate [13], and quasi-static thermoelastic solutions for a three-dimensional medium [14]. However, these studies mainly considered rectangular, circular, or other canonically shaped bodies, leaving open the problem of sharp-cornered cutouts and multiply connected domains.

The theory of R-functions, based on the operators of the algebra of logic, was created by V. L. Rvachev [15], [16] for the analytical–numerical solution of boundary value problems for bodies of complex topology, and was further developed by V. L. Rvachev, L. V. Kurpa [17], [18], [19], T. V. Shmatko, N. D. Pankratova, A. V. Sinekop, G. Barakati, and others. In their works, the combination of the Kantorovich-type solution structure of the R-function method with the Bubnov–Galerkin approach was applied mainly to isotropic and orthotropic elasticity problems, in particular to free vibration and stability problems, whereas the coupled effects of multiphysics fields were practically not taken into account.

In Uzbekistan, a scientific school on the algorithmization of problems of the theory of elasticity was established by V. K. Kabulov [20], and its traditions are being continued in the works of F. Nuraliev and co-authors on magnetoelasticity problems for thin plates of complex shape [21], [22], [23], [24], [25]. Nevertheless, an approach that simultaneously accounts for transverse shear, von Kármán nonlinearity, and the coupling of thermo-electro-magnetic fields in anisotropic plates with complex cutouts has not been sufficiently studied. This gap determines the relevance of the present research.

MATHEMATICAL MODEL AND SOLUTION METHOD

GOVERNING EQUATIONS

The thermo-magneto-elastic state of the plate is described on the basis of the Hamilton–Ostrogradsky variational principle [21]:

$$\int_t (\delta K - \delta \Pi + \delta A) dt = 0; \quad t_1 \leq t \leq t_2. \quad (1)$$

where K , Π , and A are the kinetic energy, the potential energy, and the work of external (mechanical and electromagnetic) forces, respectively. Based on the Kirchhoff–Love hypothesis, the displacement field is taken in the form

$$u_1 = u_x - z \frac{\partial w}{\partial x}, \quad u_2 = v_y - z \frac{\partial w}{\partial y}, \quad u_3 = w; \quad (2)$$

and the geometrically nonlinear strains are determined from the von Kármán-type Cauchy relations:

$$\begin{cases} \varepsilon_{xx} = \frac{\partial u}{\partial x} - z \frac{\partial^2 w}{\partial x^2} + \frac{1}{2} \frac{\partial w}{\partial x} \times \frac{\partial w}{\partial x}, \\ \varepsilon_{yy} = \frac{\partial v}{\partial y} - z \frac{\partial^2 w}{\partial y^2} + \frac{1}{2} \frac{\partial w}{\partial y} \times \frac{\partial w}{\partial y}, \\ \varepsilon_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} - 2z \frac{\partial^2 w}{\partial x \partial y} + \frac{\partial w}{\partial x} \times \frac{\partial w}{\partial y}. \end{cases} \quad (3)$$

The generalized Duhamel–Neumann thermoelastic law for an anisotropic material is applied [6]:

$$\sigma_{ij} = C_{ijkl} \varepsilon_{kl} - C_{ijkl} \alpha_{kl} (T - T_0) \quad (4)$$

Here C_{ijkl} are the elastic constants of the anisotropic material, α_{kl} is the tensor of linear thermal expansion coefficients, and $(T - T_0)$ is the temperature change.

Substituting the variations into principle (1) yields the following system of equations of motion [4]:

$$\begin{aligned}
& -\rho h \frac{\partial^2 u}{\partial t^2} - h C_{1111} \frac{\partial^2 u}{\partial x^2} - \frac{h}{2} C_{1212} \frac{\partial^2 u}{\partial y^2} - \frac{h}{2} C_{1111} \frac{\partial^3 w}{\partial x^3} - \frac{h}{2} C_{1222} \frac{\partial^3 w}{\partial y^3} - \\
& - \frac{h}{2} C_{1112} \frac{\partial^2 v}{\partial x^2} - \frac{h}{2} C_{1222} \frac{\partial^2 v}{\partial y^2} - (h C_{1122} + \frac{h}{2} C_{1212}) \frac{\partial^2 v}{\partial x \partial y} - (\frac{h}{2} C_{1122} + \frac{h}{2} C_{1212}) \frac{\partial^3 w}{\partial x \partial y^2} - \\
& - (h C_{1211} + \frac{h}{2} C_{1112}) \frac{\partial^2 u}{\partial x \partial y} - (\frac{h}{2} C_{1112} + \frac{h}{2} C_{1211}) \frac{\partial^3 w}{\partial x^2 \partial y} + \\
& + h \frac{\partial}{\partial x} (\alpha_{11} (T - T_0)) + h \frac{\partial}{\partial y} (\alpha_{12} (T - T_0)) + N_x + R_x + q_x + T_{zx} = 0, \\
& - \rho h \frac{\partial^2 v}{\partial t^2} - h C_{2222} \frac{\partial^2 v}{\partial y^2} - \frac{h}{4} C_{1212} \frac{\partial^2 v}{\partial x^2} - \frac{h}{2} C_{2212} \frac{\partial^2 u}{\partial y^2} - \frac{h}{2} C_{1211} \frac{\partial^2 u}{\partial x^2} - \\
& - \frac{h}{2} C_{2222} \frac{\partial^3 w}{\partial y^3} - \frac{h}{4} C_{1211} \frac{\partial^3 w}{\partial x^3} - (h C_{2211} + \frac{h}{4} C_{1212}) \frac{\partial^2 u}{\partial x \partial y} - (\frac{h}{2} C_{2212} + \\
& + \frac{h}{2} C_{1222}) \frac{\partial^2 v}{\partial x \partial y} - (\frac{h}{2} C_{2211} + \frac{h}{4} C_{1212}) \frac{\partial^3 w}{\partial x^2 \partial y} - (\frac{h}{2} C_{2212} + \frac{h}{4} C_{1222}) \frac{\partial^3 w}{\partial x \partial y^2} + \\
& + h \frac{\partial}{\partial y} (\alpha_{22} (T - T_0)) + \frac{h}{2} \frac{\partial}{\partial x} (\alpha_{12} (T - T_0)) + N_y + R_y + q_y + T_{zy} = 0, \\
& - \rho h \frac{\partial^2 w}{\partial t^2} - \frac{h}{2} C_{1111} (\frac{\partial u}{\partial x} \frac{\partial^2 w}{\partial x^2} + \frac{1}{2} \frac{\partial^4 w}{\partial x^4}) - \frac{h}{2} C_{1122} \frac{\partial v}{\partial y} \frac{\partial^2 w}{\partial x^2} - \\
& - \frac{h}{2} \frac{\partial^4 w}{\partial x^2 \partial y^2} (\frac{1}{2} C_{1122} + \frac{1}{2} C_{2211} + C_{1212}) - \frac{h}{4} C_{1112} \frac{\partial u}{\partial y} \frac{\partial^2 w}{\partial x^2} - \\
& - \frac{h}{2} \frac{\partial^4 w}{\partial x^3 \partial y} (\frac{1}{2} C_{1112} + C_{1211}) - \frac{h}{2} C_{2211} \frac{\partial u}{\partial x} \frac{\partial^2 w}{\partial y^2} - \frac{h}{2} C_{2222} (\frac{\partial v}{\partial y} \frac{\partial^2 w}{\partial y^2} + \\
& + \frac{1}{2} \frac{\partial^4 w}{\partial y^4}) - \frac{h}{4} C_{2212} (\frac{\partial u}{\partial y} \frac{\partial^2 w}{\partial y^2} + \frac{\partial v}{\partial x} \frac{\partial^2 w}{\partial y^2}) - \frac{h}{2} \frac{\partial^4 w}{\partial x \partial y^3} (\frac{1}{2} C_{2212} + C_{1222}) - \\
& - h C_{1211} \frac{\partial u}{\partial x} \frac{\partial^2 w}{\partial x \partial y} - h C_{1222} \frac{\partial v}{\partial y} \frac{\partial^2 w}{\partial x \partial y} - \frac{h}{2} C_{1212} (\frac{\partial u}{\partial y} \frac{\partial^2 w}{\partial x \partial y} + \frac{\partial v}{\partial x} \frac{\partial^2 w}{\partial x \partial y}) + \\
& + \frac{h}{2} \frac{\partial^2 w}{\partial x^2} \alpha_{11} (T - T_0) + \frac{h}{2} \frac{\partial^2 w}{\partial y^2} \alpha_{22} (T - T_0) + h \frac{\partial^2 w}{\partial x \partial y} \alpha_{12} (T - T_0) + \\
& + \frac{h}{12} C_{1111} \frac{\partial^4 w}{\partial x^4} + \frac{h}{12} \frac{\partial^4 w}{\partial x^2 \partial y^2} (C_{1122} + C_{2212} + C_{1211}) + \frac{h}{12} \frac{\partial^4 w}{\partial x^3 \partial y} (C_{1112} + C_{2211}) + \\
& + \frac{h}{12} \frac{\partial^4 w}{\partial x \partial y^3} (C_{2222} + C_{1212}) + \frac{h}{12} C_{1222} \frac{\partial^4 w}{\partial y^4} + N_z + R_z + q_z + T_{zz} = 0,
\end{aligned}
\tag{5}$$

Here c_v — is the specific heat capacity of the material; k_x, k_y, k_z — are the thermal conductivity coefficients; Q_s — denotes internal heat sources (heat generated by physical or electromagnetic dissipation); T_0 is the initial (reference) temperature; $\beta_{ij} = C_{ijkl} \alpha_{kl}$ is the thermoelastic coupling tensor (C_{ijkl} is the stiffness tensor and α_{kl} are the thermal expansion coefficients); $\partial \varepsilon_{ij} / \partial t$ is the strain-rate tensor (including both mechanical and nonlinear components). The normal (N_{xx}, N_{yy}) and shear (N_{xy}, N_{yx}) forces are defined per unit length of the plate midsurface; M_{xx} and M_{yy} are the bending moments, and M_{xy} is the twisting moment; q_x, q_y, q_z are the mechanical loads acting on the plate surface; p^{em} and m^m are the electromagnetic (Lorentz) force and moment terms [7], [21].

SOLUTION ALGORITHM: R-FUNCTION METHOD AND BUBNOV–GALERKIN DISCRETIZATION

System of equations (5), together with the corresponding boundary conditions, is solved by the structural R-function method [15], [17], [21]. In general form, the displacement components are sought as the following series:

$$\begin{aligned}
u(x, y, t) &= \omega(x, y) \sum_{k=1}^{N_1} b_k(t) \varphi_k(x, y), \\
v(x, y, t) &= \omega(x, y) \sum_{k=1}^{N_1} c_k(t) \varphi_k(x, y), \\
w(x, y, t) &= \omega^2(x, y) \sum_{k=1}^{N_1} a_k(t) \varphi_k(x, y),
\end{aligned}
\tag{6}$$

Here $\varphi_k(x, y)$ are algebraic basis polynomials, and $a_k(t)$ are time-dependent generalized coordinates. The in-plane u and v displacement fields vanish on the boundary Γ — of the structure but are not subject to rotational constraints. This results in a first-order structure. Here $b_k(t)$ and $c_k(t)$ are the in-plane generalized coordinates, respectively. In this case, ω is the normalized R-function of the domain; the function ω_I , constructed for the considered domain by means of R-operations, serves precisely this purpose. When the plate boundary is fully clamped, the solution structure simplifies and takes the explicit form [21]:

$$u = \omega \Phi_1, \quad v = \omega \Phi_2, \quad w = \omega^2 \Phi_3. \tag{7}$$

Spatial discretization is performed by the Bubnov–Galerkin method, which yields the following matrix system with respect to the vector of unknown coefficients $C = \{C^{(1)}, C^{(2)}, \dots, C^{(n)}\}$. [21]:

$$\ddot{A}C + BC = F. \tag{8}$$

where A and B are block matrices of mass and stiffness type, respectively (their elements, expressed as double integrals involving Φ_i , are computed by Gaussian quadrature), and F is the load vector. System (8) is integrated step by step in time using the Newmark method. The computation was carried out according to the five-stage algorithm [21] summarized in Fig. 1.

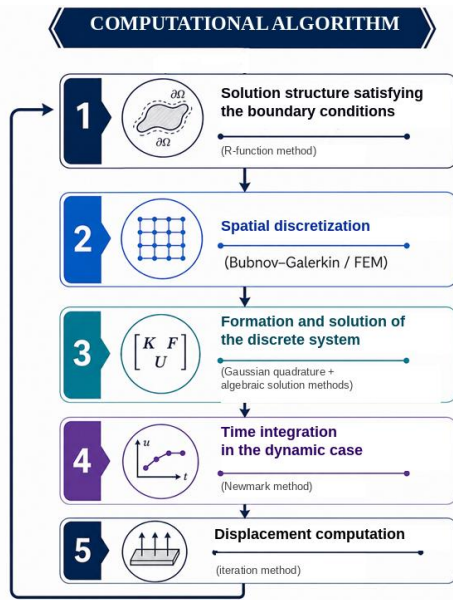


Fig. 5. Five stages of the computational algorithm based on the R-function method and the Bubnov-Galerkin method.

The algorithm includes: construction of a solution structure satisfying the boundary conditions (R-function method); spatial discretization (Bubnov-Galerkin); formation and solution of the discrete system based on Gaussian quadrature; time integration by the Newmark method; and computation of displacements from the obtained coefficients.

ANALYSIS AND RESULTS

At this stage of the study, the stress-strain state of an anisotropic thin plate with a complex (I-shaped) contour under multifactor fields, namely transverse and in-plane mechanical forces, a constant temperature field, and a stationary magnetic field, was numerically modeled.

For the computational experiment, an I-shaped composite (graphite-epoxy) plate with dimensions $a \times b = 1.0 \times 1.0$ m and thickness $h = 0.01$ m, rigidly clamped along its edges ($w = 0, \partial w / \partial n = 0$), was selected. The orthotropic properties of the material are characterized by the degree of anisotropy of its elastic moduli ($E_1 = 1.81 \cdot 10^{11}$ Pa, $E_2 = 1.03 \cdot 10^{10}$ Pa) and linear thermal expansion coefficients ($\alpha_1 = -3.0 \cdot 10^{-7} K^{-1}$, $\alpha_2 = 2.8 \cdot 10^{-5} K^{-1}$).

Computational Experiment 1: Superposition of Multifactor (Mechanical, Magnetic, and Thermal) Loads on a Transversely Isotropic Plate of Complex Shape

The use of composite and anisotropic materials in modern engineering and industry is growing rapidly. Such materials are advantageous owing to their high specific strength. However, since material properties vary with direction, accurate prediction of deformation is a challenging task. The aim of this experiment is to compute the transverse deflection of the plate and to assess its serviceability under various loads.

Object of study and boundary conditions. Within the parameters of this study, the transverse bending of a rectangular plate with rigidly clamped edges ($w = 0, \partial w / \partial n = 0$) and dimensions of 1×1 m ($a = 1$ m, mid-

section $y = 0.5$ m) was investigated. A modern anisotropic composite, unidirectional graphite-epoxy (UD lamina), was selected as the plate material, and its thermomechanical properties were taken into account. The combined effects of a surface-distributed mechanical load ($q = 100$ kPa), a magnetic field, and an elevated temperature ($\Delta T = 70$ K) on the plate were comprehensively evaluated.

The results show that in the initial calculation according to classical linear theory, the maximum deflection of the plate was $w_{\max} = 18.07$ mm. When geometric nonlinearity was taken into account, the system of nonlinear equations was solved iteratively, and full convergence was achieved after exactly 8 iterations. At the end of the 8th iteration, the actual maximum deflection was found to be $w_{\max} = 15.10$ mm.

The difference between the two approaches was 16.46%. This iterative process made it possible to correctly capture a physical phenomenon characteristic of thin composite shells and plates, namely the membrane stiffening effect: as the plate bends, the tensile stresses arising in its midsurface increase the resistance of the system to bending. Using the iterative method, the balance between the internal stresses and external loads of the system was found, and the tendency of linear theory to overestimate deformation was eliminated.

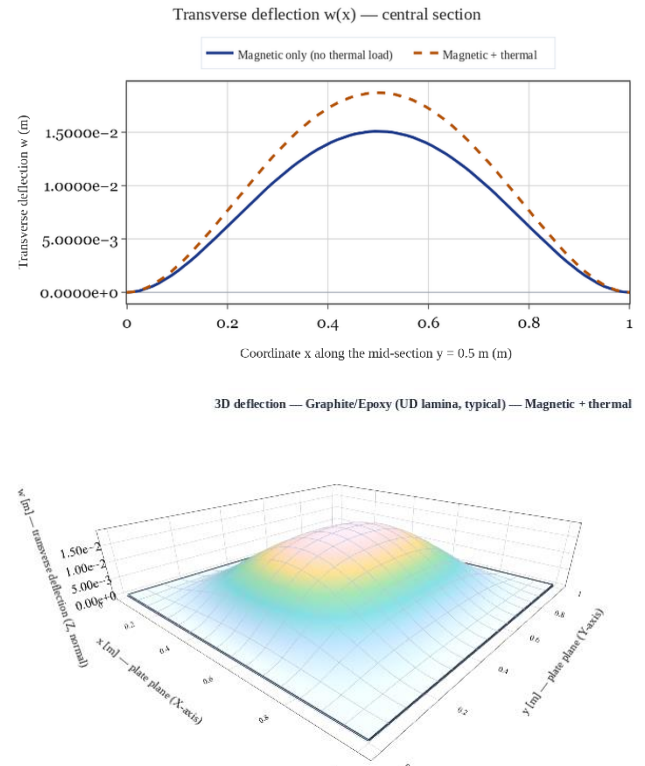


Fig. 6. Comparison of transverse deflection profiles of a plate of complex shape under a surface load, a magnetic field, and a temperature change: deflection along the central section (top) and three-dimensional deflection surface for the magnetic field and temperature case (bottom).

Combined effect of thermomagnetic loads. At the next stage of the study, a temperature increase of $\Delta T = 70$ K was added to the system (case B: magnetic field + temperature). The resulting $w(x)$ plot along the central section (solid blue and dashed orange lines in Fig. 2) clearly shows the rigid clamping condition at the edges:

both the deflection and the slope are zero at the boundaries.

Computational Experiment 2: Effect of the Thickness of an I-Shaped Composite Plate on Its Stiffness Characteristics in a Multifactor Environment

The effect of the thickness of an I-shaped orthotropic composite plate (graphite–epoxy, $\rho = 1600 \text{ kg/m}^3$, $E_1 = 181 \text{ GPa}$, $E_2 = 10.3 \text{ GPa}$) on its stress–strain state was investigated. In a multifactor environment in which, in addition to surface loads, a quasi-static magnetic field ($B = 0.30 \text{ T}$) and a temperature change ($\Delta T = 20 \text{ K}$) were applied to the system, the thickness was varied within $h \in [0.006, 0.015] \text{ m}$, and the results were subjected to correlation analysis with respect to the relationship $D \sim h^3$.

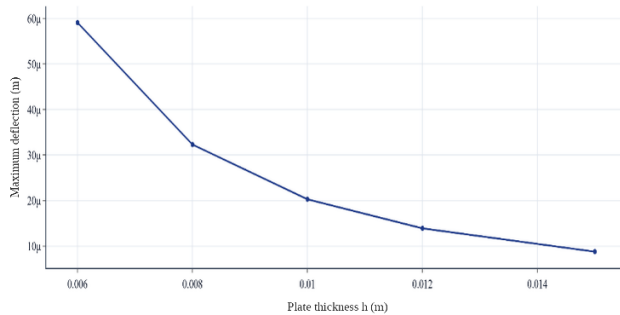


Fig. 7. Effect of the thickness of an I-shaped graphite–epoxy plate on the maximum deflection $W_{\max}$ in a multifactor environment.

At $h = 0.006 \text{ m}$ the maximum deflection is approximately $60 \mu\text{m}$; at $h = 0.010 \text{ m}$ it drops sharply to approximately $20 \mu\text{m}$, and at $h = 0.015 \text{ m}$ it stabilizes below $10 \mu\text{m}$. The Pearson correlation coefficient between $w_{\max}$ and $1/h^3$ is $R = 0.996$, which confirms that the developed model obeys the fundamental laws of elasticity even in multifactor coupled problems.

CONCLUSIONS

The multiparametric numerical studies performed have shown that the model and algorithm based on the R-function method and the Bubnov–Galerkin variational approach are efficient and reliable for describing the nonlinear deformation of anisotropic plates of complex shape in thermo-magneto-electro-elastic fields: in comparison with a classical benchmark problem, the maximum relative error did not exceed 0.81%, which allows the approach to be used in engineering calculations.

It was established that geometric nonlinearity must be taken into account for composite panels operating under intense surface loads and temperature changes: linear theory overestimates the deflection at large displacements, whereas the nonlinear model reveals the self-stiffening effect caused by membrane stresses and prevents excessive material consumption. The fact that the inverse-cubic dependence of the maximum deflection on thickness persists in a multifactor environment ($R = 0.996$) indicates that the optimal choice of thickness is the principal means of reducing structural mass.

Under static loading, the magnetic field reduces the effective stiffness of the plate through Lorentz forces, whereas in the dynamic regime it acts as equivalent damping and shortens the transient stage of vibrations; this result provides a basis for developing contactless active vibration control systems in smart structures. The macro-topology and boundary conditions of the plate fundamentally change the deformation field: I- and T-shaped plates with rigidly clamped inner contours retain high stiffness while reducing mass, whereas leaving the outer edges free increases the deflection by more than 325%. Therefore, rigid clamping of the outer contours of load-bearing composite elements is recommended.

For canonical domains, the Bubnov–Galerkin method with a low-order trigonometric basis yields cost-effective results, whereas for domains with complex cutouts it is advisable to use a solution structure based on R-functions or fine-mesh methods. Future research is recommended to be extended to experimental verification, laminated composites, and parametric resonance problems.

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