

ON FIXED POINTS OF HAMMERSTEIN OPERATOR WITH DEGENERATE KERNEL

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We construct a degenerate kernel such that Hammerstein operator has exactly three positive fixed points.

Let $\varphi_1(u), \varphi_2(u)$ and $\psi_1(t), \psi_2(t)$ be positive functions from from $C_0^+[0, 1] := \{f \in C[0, 1 : f(x) \geq 0]\} \setminus \{f(x) \equiv \theta\}$. We consider special type of Hammerstein operator H with degenerate kernel (see [1]):

$$(Hf)(t) = \int_0^1 (\varphi_1(u)\psi_1(t) + \varphi_2(u)\psi_2(t))f^2(u)du.$$

We consider symmetrical matrix 2×2 $A = (a_{ij})_{i,j=1,2}$ and $B = (b_{ij})_{i,j=1,2}$ with (strictly) positive elements. For matrix A and B we define quadratic operator (QO) Q in cone of the space R^2 by the rule:

$$Q(x, y) = (\alpha_{11}x^2 + 2\alpha_{12}xy + \alpha_{22}y^2, \beta_{11}x^2 + 2\beta_{12}xy + \beta_{22}y^2).$$

Denote

$$\alpha_{11} = \int_0^1 \psi_1(t)\varphi_1^2(u)du > 0, \quad \alpha_{12} = \int_0^1 \psi_1(t)\varphi_1(u)\varphi_2(u)du > 0,$$

$$\alpha_{22} = \int_0^1 \psi_1(t)\varphi_2^2(u)du > 0;$$

$$\beta_{11} = \int_0^1 \psi_2(t)\varphi_1^2(u)du > 0, \quad \beta_{12} = \int_0^1 \psi_2(t)\varphi_1(u)\varphi_2(u)du > 0,$$

$$\beta_{22} = \int_0^1 \psi_2(t)\varphi_2^2(u)du > 0.$$

Clearly, an arbitrary nontrivial positive fixed points of the QO Q is strictly positive (see [2]).

Lemma [3]. Following statements hold:

- (1) If the point $\omega = (x_0, y_0)$ is a strictly fixed point of QO Q , then $\xi_0 = \frac{y_0}{x_0}$ is a root of the cubic algebraic equation

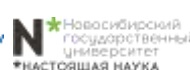
$$\alpha_{22}\xi^3 + (2\alpha_{12} - \beta_{22})\xi^2 + (\alpha_{11} - 2\beta_{12})\xi - \beta_{11} = 0.$$

- (2) If the point $\omega = (x_0, y_0) \in R_2^+$ is a fixed point of QO Q , then $\omega \in R_2^>$ and $\xi_0 = \frac{y_0}{x_0}$ is a root of the cubic algebraic equation

$$\alpha_{22}\xi^3 + (2\alpha_{12} - \beta_{22})\xi^2 + (\alpha_{11} - 2\beta_{12})\xi - \beta_{11} = 0.$$

We define two continuous positive functions $\varphi_1(u)$ and $\varphi_2(u)$ on $[0, 1]$

$$\varphi_1(u) = \begin{cases} u + \frac{1}{16}, & \text{if } u \in \left[0, \frac{1}{4}\right] \\ \frac{9}{16} - u, & \text{if } u \in \left[\frac{1}{4}, \frac{1}{2}\right] \\ \frac{1}{16}, & \text{if } u \in \left[\frac{1}{2}, 1\right] \end{cases}$$



$$\text{and}$$

$$\varphi_2(u) = \begin{cases} \frac{1}{16}, & \text{if } u \in \left[0, \frac{1}{2}\right] \\ u - \frac{7}{16}, & \text{if } u \in \left[\frac{1}{2}, \frac{3}{4}\right] \\ \frac{17}{16} - u, & \text{if } u \in \left[\frac{3}{4}, 1\right] \end{cases}$$

We define two continuous positive functions $\psi_1(t)$ and $\psi_2(t)$ on $[0, 1]$:

$$\psi_1(t) = \begin{cases} 260 - 512t, & \text{if } t \in \left[0, \frac{1}{2}\right], \\ 4, & \text{if } t \in \left[\frac{1}{2}, 1\right]. \end{cases}$$

$$\psi_2(t) = \begin{cases} 4, & \text{if } t \in \left[0, \frac{1}{2}\right] \\ 512t - 252, & \text{if } t \in \left[\frac{1}{2}, 1\right] \end{cases}$$

Denote

$$K(t, u) = \varphi_1(u)\psi_1(t) + \varphi_2(u)\psi_2(t), \quad t, u \in [0, 1].$$

Theorem. Let $a, b \in R^+$. Then there exist exactly three positive fixed points of the operator H_2 with the kernel $K(t, u)$.

Reference

- [1]. Krasnosel'ski, M.A.: Positive solutions of operator equations. (Gos. Izd. Moscow, 1969) (Russian).
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- [3]. Yu. Kh. Eshkabilov, Sh. D. Nodirov, F. H. Haydarov: Positive fixed points of quadratic operators and Gibbs Measures, Positivity 20(4) (2016).

